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Estimation of diurnal global solar irradiance on a horizontal surface—new approach

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Abstract : A method is suggested to predict the hourly global solar irradiance incident on a horizontal surface, in terms of the length of the day and the maximum value of incident solar irradiance at midday. The obtained distribution can be easily integrated along the length of the day to get the daily totals of solar irradiance Comparison between computed and published experimental data from Barcelona, Hong Kong, Egypt, Saudi Arabia is also made

Keywords : Global solar irradiance, horizontal surface, theoretical prediction

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1. Introduction

The prediction of the global solar radiation on a horizontal surface at a certain location on the earth's surface is very important in designing and estimation of the performance of systems utilizing solar energy at the considered site.

Much work has been done [1–26] based principally on empirical formulae—to evaluate the received solar radiation and its variation during different intervals of calender time. Most of such trials are made to predict the monthly and annual daily global radiation received on a horizontal surface. In such trials, various parameters are used to predict quantitatively the incident solar radiation such as the solar constant, the sunshine hours, the solar declination angle, the latitude, the sunset hour angle, the relative humidity, the air temperature, cloud cover, the number of raining days in month, *etc*.

Several attempts have been made earlier [5,12,20] to predict the hourly values of the total solar radiation $q(t)$ W/m^2, where t is the local time in hours. Such attempts either require a long term meteorological data or are not easily integrated analytically along the length of the day.

The present paper presents a new approach to predict the required distribution function $q(t)$ W/m^2, in a form that avoids above mentioned limitations. This in turn is very important, not only for the design but also for the efficiency evaluation of the solar energy conversion systems.

2. Derivation of the basic equations

The experimental measurements for the diurnal variation of the total global solar radiation $q(t)$ for clear days as published by different authors [8,12–15] revealed that the corresponding curves exhibit the same feature. Namely, a symmetrical distribution about a maximum value q_0 occurring at the mid time $t_0 = \frac{t_d}{2}$, between sunrise t_r and sunset t_s, where, t is the local time in hours and $t_d = (t_s - t_r)$ is length of the considered day.

The distribution function $q(t)$ for the hourly average global intensity on a horizontal surface may be approximated by a quadratic expression in the form :

$$q(t) = a_0 + a_1|t - t_0| + a_2|t - t_0|^2. \tag{1}$$

Eq. (1) is subjected to the following conditions :

i) At $t = t_0$, $q(t) = q_0$, (2)

and $\left.\frac{\partial q}{\partial t}\right|_{t=t_0} = 0$; (3)

ii) at $t = t_r$, $q(t) = 0$; (4)

iii) at $t = t_s$, $q(t) = 0$. (5)

Applying condition (2) to eq. (1), we get

$$a_0 = q_0. \tag{6}$$

Applying condition (3) to eq. (1), we get

$$a_1 = 0. \tag{7}$$

Since $|t_r - t_0| = |t_s - t_0 = \frac{t_d}{2}$, one finds that either (5) or (6) with eq. (1) gives the same result, that is :

$$a_2 = -\frac{4q_0}{t_d^2}. \tag{8}$$

Finally, one can write the suggested distribution in the form :

$$q(t) = q_0\left\{1 - 4\left|\frac{t - t_0}{t_d}\right|^2\right\}. \tag{9}$$

When the obtained distribution eq. (9) is used to fit some of the published data [13,14] on the hourly values of the total solar radiation, one gets unsatisfactory results. The experimental and calculated distributions coincide at the points t_r, t_s and t_0. Small deviation between the two distributions at the intermediate points is recorded. Thus, one has to modify the suggested expression (1) to be in the form :

$$q(t) = q_0 \left\{ 1 - 4 \quad t - t_r \right. \qquad \cdot F(t), \tag{10}$$

where $F(t)$ is a correction function that must satisfy the following conditions :

$$\begin{array}{llllll} \text{(i)} & 0 \le & F(t) & \le 1, & & \\ \text{(ii)} & F(t) & = 0 & \text{at} & t = t_r, & \\ \text{(iii)} & & = 0 & \text{at} & t = t_s, & \\ \text{(iv)} & & = 1 & \text{at} & t = t_0. & \end{array} \tag{11}$$

To meet such requirements, the function $F(t)$ is chosen to be in the form :

$$F(t) = \frac{4(t - t_r)\,(t - t_s)}{} \tag{12}$$

In the light of the above considerations, the modified distribution is written in the form :

$$q(t) = q_0 \left\{ 1 - \quad 4(t - t_0)^2 \qquad 4(t - t_r)\,(t - t_s) \right. \tag{13}$$

The obtained distribution eq. (13) is used to fit some of the published experimental data on the average hourly total solar irradiance on a horizontal surface at :

(1) Barcelona (Spain, 1973) located at 41° 23′ N, 2° 7′ E during following months : June, September and December [13].

(2) Hong Kong (1979) located at 22° 19′ N, 114° 10′ E during following months : January, April, July and November [14].

(3) Al Ahram (Egypt, 1980) located at 23° 58′ N for two chosen months : July and September [27].

(4) Jeddah (Saudi Arabia, 1982) located at 21° 37′ N, 40° 25′ E on 10 April [16].

The published data and the corresponding computed values according to the suggested model eq. (13) for the four cities are given in Table 1 for Barcelona, in Table 2 for Hong Kong, in Table 3 for Al Ahram and in Table 4 for Jeddah.

Table 1. Average hourly values of the total irradiance (W/m^2) in Barcelona, Spain (years 1973–1975)

Month	Solar time (hours)	Shifted time (hours)	$q_{exp.}$	$q_{calc.}$
January	4	0.0	0.0	0.0
September	5	0.0	0.0	0.0
December	7	0.0	0.0	0.0
January	5.5	1.5	75.8	82.0006
September	7.5	2.5	183.0	197.94
December	7.5	0.5	9.6	11.19
January	6.5	2 5	198.3	197.449
September	8.5	3.5	317.6	323.437
December	8.5	1.5	83.8	80.63
January	7.5	3.5	338.0	331.7833
September	9.5	4.5	430.7	437.671
December	9.5	2.5	189.4	174.38
January	8.5	4.5	471.5	464.2149
September	10.5	5.5	505.3	523.406
December	10.5	3.5	260.5	256.71
January	9.5	5 5	577.1	578.0992
September	11.5	6.5	574.7	569.1476
December	11.5	4.5	302.5	303.83
January	10.5	6.5	681.7	660.9556
September	12.0	7.0	575.0	575.0
December	12.0	5.0	306.0	310.0
January	11.5	7.5	707.8	704.5
September	12.5	7.5	567.1	569.147
December	12.5	5.5	297.1	303.83
January	12.0	8.0	710.0	710.0
September	13.5	8.5	518.8	523.406
December	13.5	6.5	257.0	256.71
January	12.5	8.5	700.8	704.5
September	14.5	9.5	426.8	437.671
December	14.5	7.5	188.4	174.38
January	13.5	9.5	661.8	660.9557
September	15.5	10.5	305.2	323.437
December	15.5	8.5	87.7	80.630
January	14.5	10.5	580.3	578.0992
September	16.5	11.5	174.0	197.94
December	16.5	9.5	11.1	11.191
January	15.5	11.5	456.6	464.2149
September	19.0	14.0	000.0	000.0000
December	17.0	10.0	000.0	000.0000
January	16.5	12.5	324.4	331.7833
September	00.0	00.0	000.0	000:0000
December	00.0	00.0	000.0	000.0000

Table 1. *(Cont'd.)*

Month	Solar time (hours)	Shifted time (hours)	$q_{exp.}$	$q_{calc.}$
January	17.5	13.5	184.1	197.4449
September	00.0	00.0	000.0	000.0000
December	00.0	00.0	000.0	000.0000
January	18.5	14.5	60.7	82.0006
September	00.0	00.0	000.0	000.0000
December	00.0	00.0	000.0	000.0000
January	20.0	16.0	00.0	000.0000
September	00.0	00.0	000.0	000.0000
December	00.0	00.0	000.0	000.0000

Table 2. Monthly average hourly total irradiance (MJ $m^2 hr^{-1}$) received on a horizontal surface in Hong Kong, Jan. 1979.

Month	Solar time (hours)	Shifted time (hours)	q_{Exp}	$q_{Calc.}$
January	6.0	0.0	0.0	0.0
April	5.0	0.0	0.0	0.0
July	5.0	0.0	0.0	0.0
November	6.0	0.0	0.0	0.0
January	6.5	0.5	0.07	0.0428
April	5.6	0.5	0.03	0.0309
July	6.5	1.5	0.41	0.4164
November	6.5	0.5	0.04	0.04
January	7.5	1.5	0.33	0.3215
April	7.5	2.5	0.43	0.561
July	7.5	2.5	0.9	0.979
November	7.5	1.5	0.29	0.3017
January	8.5	2.5	0.75	0.7312
April	8.5	3.5	0.77	0.9169
July	8.5	3.5	1.5	1.5975
November	8.5	2.5	0.74	0.8052
January	9.5	3.5	1.2	1.147
April	9.5	4.5	1.14	1.241
July	9.5	4.5	2.14	2.1617
November	9.5	3.5	1.26	1.2634
January	10.5	4.5	1.55	1.4766
April	10.5	5.5	1.44	1.4928
July	10.5	5.5	2.62	2.5051
November	10.5	4.5	1.61	1.6260
January	11.5	5.5	1.60	1.6567
April	11.5	6.5	1.56	1.61
July	11.5	6.5	2.02	2 81109
November	11.5	5.5	1.01	1.8244

Table 2. *(Cont'd.)*

Month	Solar time (hours)	Shifted time (hours)	$q_{Exp.}$	$q_{Calc.}$
January	12.0	6.0	1.68	1.68
April	12.0	7.0	1.63	1.63
July	12.0	7.0	2.84	2.84
November	12.0	6.0	1.85	1.85
January	12.5	6 5	1.57	1.6567
April	12.5	7.5	1.63	1.61
July	12.5	7.5	2.808	2.811
November	12.5	6.5	1.81	1.824
January	13.5	7.5	1 40	1.476
April	13.5	8.5	1.47	1.4928
July	13.5	8.5	2.6	2.5057
November	13.5	7.5	1.50	1.6259
January	14.5	8.5	1.14	1.147
April	14.5	9.5	1.43	1.241
July	14.5	9.5	2.17	2.1617
November	14.5	8.5	1.25	1.2633
January	15.5	9.5	0.72	0.3712
April	15.5	10.5	0.79	0.9166
July	15.5	10.5	1.65	1.5975
November	15.5	9.5	0 76	0.8052
January	16.5	10.5	0.30	0.3215
April	16.5	11.5	0.48	0.5611
July	16.5	11.5	1.14	0.979
November	16.5	10.5	0.28	0.3017
January	17.5	11.5	0.06	0.0428
April	17.5	12.5	0.18	0.2386
July	17.5	12.5	0.56	0.4164
November	18.0	00.0	0.00	0.0000
January	18.0	12.0	0.00	0.0000
April	19.0	14.0	0.00	0.0000
July	19.0	14.0	0.00	0.0000
November	—	—	—	—

Table 3. Global solar radiation intensity (W/m^2) received on a horizontal surface in Al-Ahram, Egypt, September 1980.

Month	Solar time (hours)	Shifted time (hours)	q_{exp}	$q_{calc.}$
September	5	0	000.0	000.000
July	5	0	000.0	000.000
September	7	2	205.0	218.309
July	6	1	115.0	73.906
September	8	3	420.0	412.740
July	7	2	330.0	251.895

Table 3. *(Cont'd.)*

Month	Solar time (hours)	Shifted time (hours)	q_{exp}	$q_{calc.}$
September	9	4	600.0	606.414
July	8	3	560.0	476.239
September	10	5	760.0	767.490
July	9	4	760.0	699 708
September	11	6	855.0	873.236
July	10	5	920.0	885.569
September	12	7	910.0	910.000
July	11	6	1010.0	1007.580
September	13	8	860.0	873.236
July	12	7	1045.0	1050.000
September	14	9	750.0	767 49
July	13	8	990.0	1007.580
September	15	10	580 0	606.414
July	14	9	880.0	885 569
September	16	11	470.0	412.740
July	15	10	755.0	699 708
September	17	12	160.0	218.309
July	16	11	530 0	476 239
September	19	14	000.0	000.000
July	17	12	330.0	251.895
September	—	—	—	—
July	18	13	110.0	73.906
September	—	—	—	—
July	19	14	000.0	000.000

Table 4. Diurnal global solar irradiance (W/m^2) received on a horizontal surface in Jeddah, Saudi Arabia, April 1982.

Solar time (hours)	Shifted time (hours)	$q_{exp.}$	$q_{calc.}$
6.08	0.00	0.00	0.00
7.25	1.17	110.00	103.871
8.25	2.17	325.00	297.676
9.25	3.17	535.00	519.258
10.25	4.17	715.00	717.892
11.25	5.17	850.00	856.857
12.25	6.17	910.00	913.434
13.25	7.17	887.50	878.908
14.25	8.17	802.50	758.564
15.25	9.17	640.00	571.990
16.25	10.17	376.80	351.577
17.25	11.17	148.30	145.517
18.25	12.17	13.60	14.807
18.66	12.58	00.00	00.000

The degree of fitting between the computed and experimental published data is expressed as percentage error *i.e.*

$$\varepsilon = \frac{q_{\text{exp}} - q_{\text{cal}}}{q_{\text{exp}}} \times 100\ \%$$

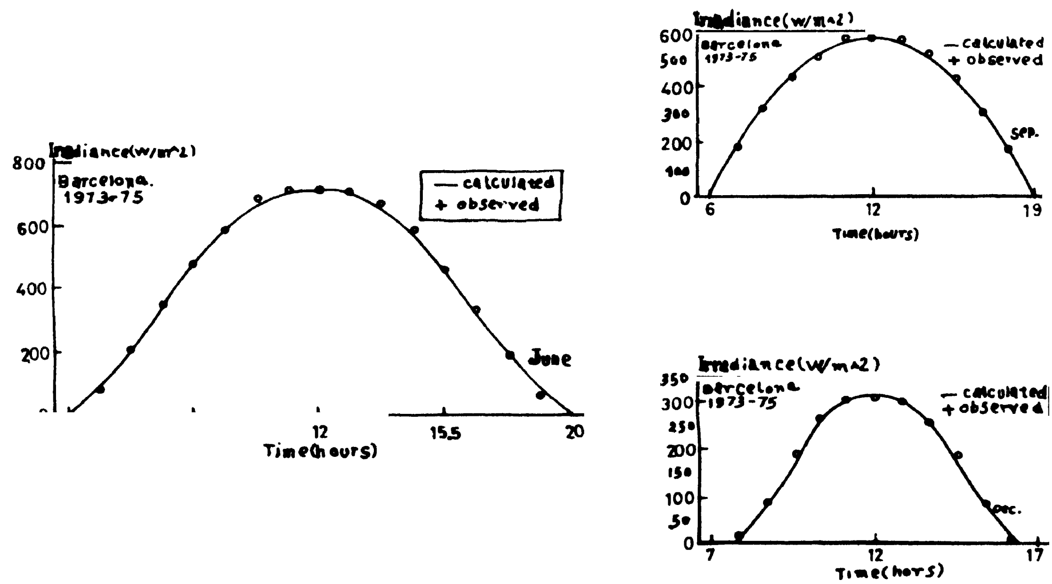

Figure 1. Observed and calculated data for Barcelona, Spain.

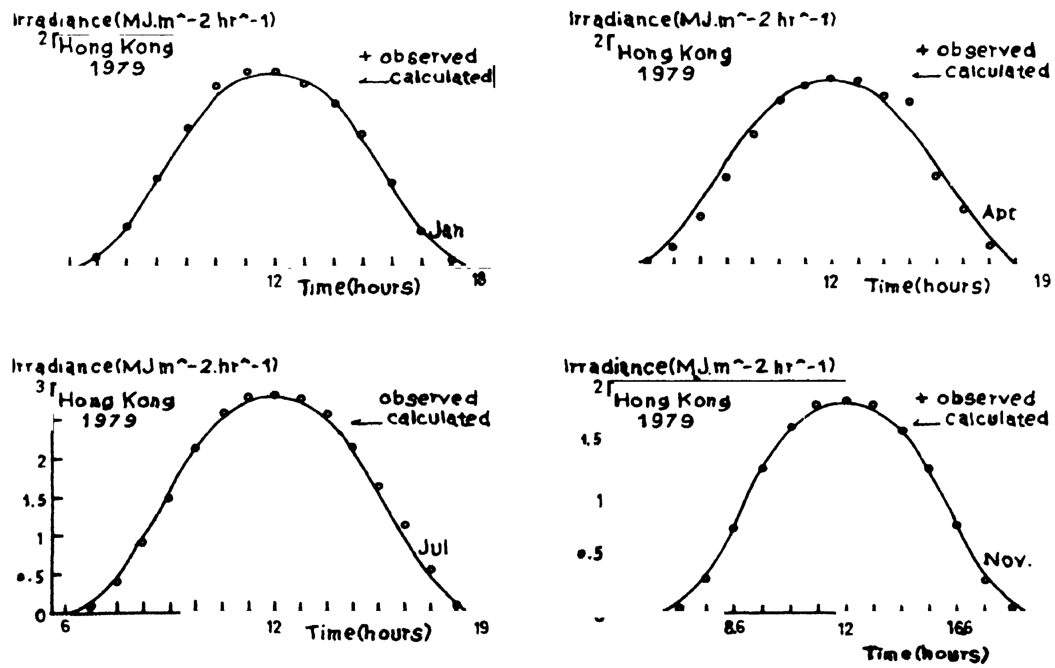

Figure 2. Observed and calculated data for Hong Kong.

Computations revealed that ε does not exceed 16% except at the two extreme limits of the curve $q(t)$.

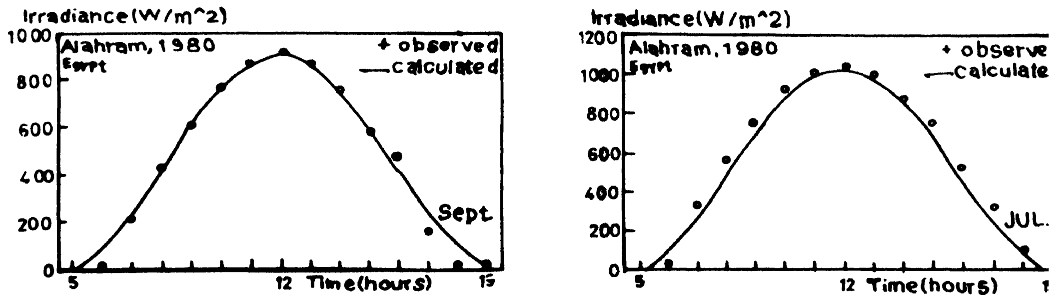

Figure 3. Observed and calculated data for Al Ahram, Egypt.

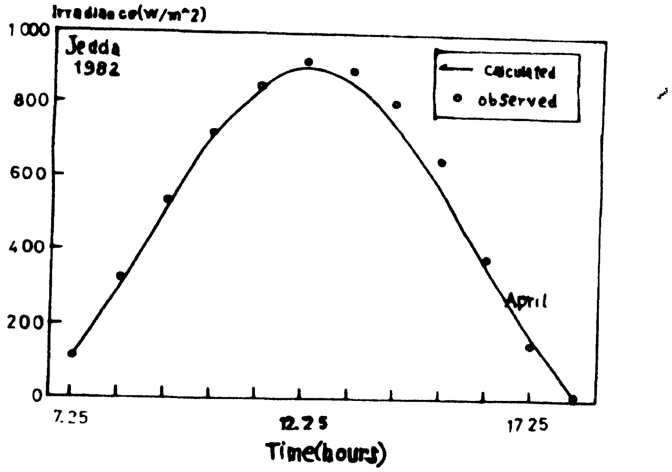

Figure 4. Observed and calculated data for Jeddah, Saudi Arabia

The experimental and computed distributions using eq. (13) are also illustrated graphically in Figures 1–4 for comparison.

3. Estimation of the average daily totals of the global solar irradiance

The daily totals of the global solar irradiance on a horizontal surface for a certain day can be obtained by integrating expression (13) over the day length t_d, this gives :

$$I = \int^{t_d} q(t)\, dt = 0.533\, q_0\, t_d. \tag{14}$$

Eq. (14) is used to fit the published data on the monthly average of hourly total irradiance received on a horizontal surface in Hong Kong [14] in the interval from December 1978 up to 1979, where, the length of the day t_d and the maximum value of the incident daily solar irradiance are obtained from the experimental published data [14].

The obtained computed values are given in Table 5 together with the corresponding maximum, minimum and mean experimental values for comparison.

Table 5. Average daily totals of global solar irradiance on a horizontal surface ($Mj\,m^{-2}\,hr^{-1}$), Hong Kong.

Month	Mean	Maximum	Minimum	Calculated Eq. (14)
J	11.48	15.29	7.03	9.79
F	11.76	15.68	9.25	8.57
M	11.95	20.16	6.87	6.11
A	13.87	17.03	12.07	11.23
M	16.13	18.55	14.23	11.44
J	17.06	21.60	14.23	15.02
J	18.84	22.52	14.55	19.43
A	18.16	22.65	14.19	12.59
S	17.02	19.70	13.90	14.25
O	15.33	19.32	11.84	19.21
N	13.87	19.67	9.13	10.55
D	12.51	14.68	10.32	12.53
Year	14.84	16.51	13.05	12.56

4. Conclusion

The comparison between the experimental and computed values shows a good agreement. This indicates that the suggested distribution is promising to predict the hourly and average daily totals global solar irradiance incident on a horizontal surface.

Moreover, the values of t_d and q_0 have to be computed on theoretical basis. Further, experimental data sets are necessary to test the degree of accuracy of the suggested model.

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